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STUDENT VERSION
Monod's Bacterial Growth Model

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Abstract: Students model growth of bacteria *E. coli* in a limiting nutrient environment using data from a historical study.

SCENARIO DESCRIPTION

Time (hr)	E. coli	Substrate
0.0	15.5	151.0
0.54	23.0	123.0
0.90	30.0	105.0
1.23	38.8	75.0
1.56	48.5	43.5
1.95	58.3	14.5
2.33	61.3	3.5
2.70	62.5	0.5

Table 1. Data pairs with units from Monod's original experiments (time in hours, biomass concentration of *E. coli* where one unit is approximately 0.75 mg dry weight/L, concentration of the growth limiting nutrient (or substrate) lactose in mg/L).

We show Monod's data in Table 1 and plots of the data of *Escherichia coli* (*E. coli*) biomass concentration in Figure 1 and growth limiting nutrient concentration in Figure 2, both as a function of time, from the 1949 study [2] by J. Monod.

> "Monod's bacterial growth model offered in 1949 [2] defines the relationship between specific growth rate and concentration of the limiting nutrient, and the relation between nutrient utilization rate and bacterial growth where $x(t)$ is the biomass concentration at time t, $s(t)$ is the concentration of the growth limiting nutrient (or substrate) at time t."[1]

One way to model the consumption of nutrients ($s(t)$) by bacteria ($x(t)$) is given by (1), a simple (and unlimited) predation model.

$$\begin{aligned} x'(t) &= \mu_m x(t)s(t) \qquad (1)\\ s'(t) &= \frac{x'(t)}{Y} \end{aligned}$$

This model has the disadvantage that one unit of bacterial concentration ($x(t)$) can consume nutrients at a rate proportional to the nutrient level, meaning per unit time we are modeling our bacteria consumption to grow in an unlimited fashion - highly unrealistic. Below is the unlimited growth model.

We offer up Monod's model [2], first published in 1949, which introduced a satiation or limiting consumption rate term $\frac{s(t)}{K_m+s(t)}$ to replace the unlimited consumption rate term found in (1), leading to

$$\begin{aligned} x'(t) &= x(t)\mu_m \frac{s(t)}{K_m + s(t))} \qquad (2)\\ s'(t) &= \frac{x'(t)}{Y} \end{aligned}$$

where

- $x(t)$ is the biomass concentration (one unit is approximately 0.75 mg dry weight/L) of *E. coli* at time t hours;
- $s(t)$ is the concentration of the growth limiting nutrient (or substrate) lactose in mg/L;
- K_m is the substrate concentration which supports half-maximum *E. coli* specific growth rate;
- μ_m is the maximum specific growth rate; and
- Y is the yield conversion coefficient.

Indeed, from a plot of Monod's data [2] on the growth of *E. coli* shown in Figure 1 we see that growth is NOT unlimited, but rather levels off and stays that way when the nutrient levels ($s(t)$) become less and less.

Assignment

1. From the data and solutions of the differential equations (possibly numerical solutions) estimate the best parameters K_m and μ_m in (2). Hint: Use second equation to reduce problem to one differential equation in one variable, $x(t)$.

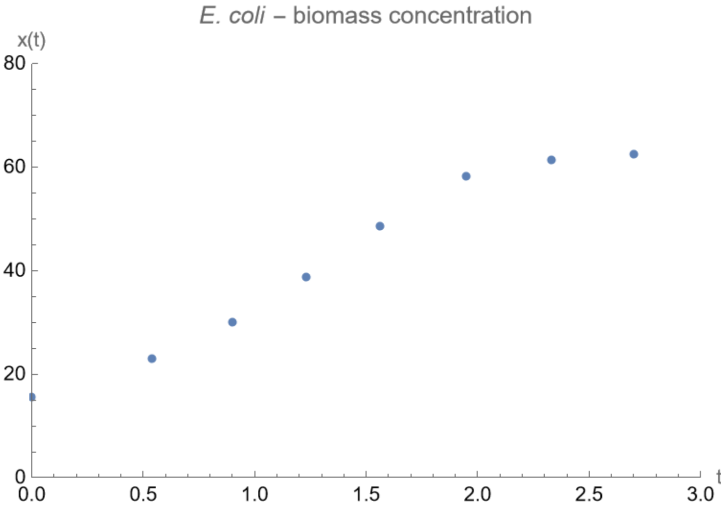

Figure 1. Plot of *E. coli* ($x(t)$) biomass concentration where one unit is approximately 0.75 mg dry weight/L.

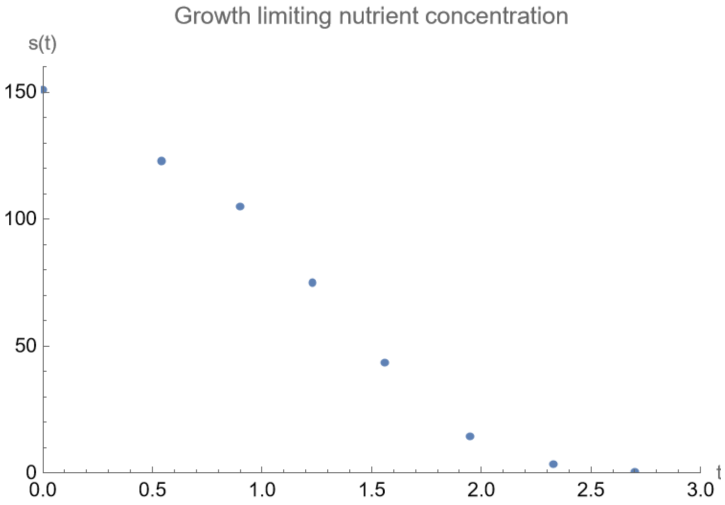

Figure 2. Plot of concentration of the growth limiting nutrient (or substrate) lactose ($s(t)$) in mg/L)

2. Show that the nutrient concentration level, K_m, is actually the level at which the *E. coli* feeds at one-half its maximum feeding rate. This gives some context and meaning to this parameter in the model and can be measured in a laboratory as well as estimated in a process such as this activity.
3. Use the parameters you obtain for (2) and show how well your model predicts the *E. coli* concentration when compared to the data for *E. coli* over time as displayed in Table 1.
4. Use the parameters you obtain for (2) and show how well your model predicts the concentration of the growth limiting nutrient when compared to the data for *E. coli* over time.

REFERENCES

[1] Lobry, J. R. and J. P. Flandrois. 1990. Comparison of estimates of Monod's growth model parameters from the same data set. *BINARY*. 3: 20-23.

[2] Monod, J. 1949. The growth of bacterial cultures. *Annual Review of Miocrobiology*. 3: 371-394.